HEAT AND MASS TRANSFER IN PACKED BEDS

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Eddy mass diffusivities, effective thermal conductivities, and wall heat transfer coefficients were measured in an 8-in. tube packed with ½- and ¾-in. glass spheres. Superficial mass velocities ranged from 110 to 1,640 lb./(hr.) (sq. ft.), corresponding to modified Reynolds numbers of 100 to 2,000. Air was the main stream fluid in all cases.

The modified Peclet group (D_pV/E^*_{td}) was found to be constant at a value of about 12 in the region of fully developed turbulence. At lower Reynolds numbers this group varied with the flow rate. Effective thermal conductivities were correlated by an equation. Modified Peclet numbers for heat transfer were about 25% less than those for mass transfer. The wall heat transfer coefficient varied with the superficial mass velocity as $h_w = 0.090(G_0^{0.75})$.

An explanation is suggested for the similarity in velocity dependence between these values and those for turbulent flow in an empty tube, based on channeling at the wall.

Among the recent advances in chemical engineering has been the development of analytic methods for the rational design of nonadiabatic, fixed-bed, catalytic converters. To apply these methods, a knowledge of radial heat and mass transport rates in the system is required. Temperatures and concentrations must be known as functions of the position in the bed in order to allow for their effects on the reaction rate.

The only reported studies of lateral mass transport rates in flowing fluid-packed-bed systems are those of Bernard and Wilhelm (*2*) and Latinen(*11*). The former measured eddy-diffusion rates for 3/8-in. alumina spheres in an 8-in. column with air as the main stream fluid and for several sizes of cylinders and spheres in a 2-in. tube with water as the main stream fluid. Latinen extended the range of this work to the water-solids system. All data were taken with the bed at a uniform temperature, and the results were expressed as modified Peclet numbers.

Earlier studies (*8, 12* to *14, 17*) on heat transfer in packed beds were confined to measuring boundary temperatures, and results were reported as over-all thermal conductivities or over-all heat transfer coefficients. These values incorporate the resistance to heat transfer within the bed as well as the heat transfer resistance at the walls of the confining tube and thus do not provide a basis for predicting radial-temperature gradients.

Radial heat transfer rates have been obtained from measured radial temperature traverses by Coberly and Marshall(*5*), Felix and Neill (*6*), Campbell and Huntington(*4*), Irvin, Olson, and Smith(*9*), Bunnell, Irvin, Olson, and Smith(*3*), and Schuler, Stallings, and Smith (*15*). Results were reported in terms of effective thermal conductivities of the gas-solid bed. It was also noted that the resistance to heat transfer in a bed increased near the confining walls. Coberly and Marshall, Felix and Neill, and Campbell and Huntington account for this by assuming k_e to be constant within the bed and postulate a heat transfer coefficient at the wall. Irvin *et al.*, Bunnell *et al.*, and Schuler and Smith handle this problem by decreasing the effective thermal conductivity values near the wall.

Baron(*1*), and Latinen(*11*) analyzed turbulent diffusion in packed beds on the basis of a "random walk" theory. Their analysis indicates that when turbulence is fully developed, the Peclet numbers for heat and mass transfer should be equal and constant at a value of about 11. Results of the previous investigations, however, have shown Peclet numbers for heat transfer to be less than those for mass transfer. There has been some doubt as to whether this discrepancy arises from differences in experimental techniques or from differences in bed-temperature conditions encountered in the two types of measurements. The present investigation has been conducted with the following objectives: (1) to extend isothermal-eddy-mass-diffusion measurements for a gas-solids system and to compare these values with those measured in the presence of a temperature gradient; (2) to measure eddy-mass diffusivities and effective thermal conductivities under identical temperature, bed, and flow conditions to permit a comparison of Peclet numbers for heat and mass transfer; and (3) to study the wall effect in greater detail.

In a cylindrical nonadiabatic fixed-bed catalytic converter, heat and mass move from regions of high concentrations to regions of low concentrations through the in-

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terstices of the bed. In the ranges of flow encountered in industrial practice, longitudinal or axial transfer occurs primarily by the mechanism of velocity transport; that is, heat and mass are carried by the net mass flow of the fluid. Since it is unlikely that there is an appreciable net mass flow of fluid in the radial direction, radial transport is believed to result primarily from interstitial fluid mixing. Evaluating lateral transport parameters from an analysis of this mixing process, however, has met with only limited success as the fluid mechanics of flow in a packed bed has not as yet been developed.

A less rigorous but more successful approach to this problem has been to treat the system as though it were a homogeneous material and incorporate all transport mechanisms except mass flow into an over-all diffusion mechanism. This assumes that the presence of the packing need be considered only insofar as it affects the numerical value of the diffusion coefficient. Although there is no theoretical basis for this procedure, it is acceptable if the over-all diffusion coefficient is assumed to be a function of all the variables that affect the actual transfer mechanisms operating in the bed.

Employing this concept, one can write a single equation to describe both heat and mass transfer in a packed bed. In cylindrical coordinates this equation is

$$V \frac{\partial f}{\partial z} = E_o^* \left(\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial z^2} \right) \quad (1)$$

The assumptions inherent in Equation (1) are

1. The bed is isotropic; i.e., E_o^* in the radial direction is equal to E_o^* in the axial direction.
2. E_o^* and V do not vary with radial position or bed depth.
3. Radial symmetry prevails.
4. The velocity in the radial direction is zero.
5. The fluid properties are constant.
6. The process is at steady state.
7. No chemical reaction is occurring.

In the case of mass transfer where natural convection is negligible, the total diffusivity is the sum of two diffusion processes operating in parallel. One is molecular diffusion in the fluid phase, and the other is a diffusion process arising from turbulent mixing in the bed interstices, known as eddy diffusion.

Equation (1) has been integrated (*2*) for the case of mass diffusion from a point source into a flowing stream of infinite extent:

$$\frac{C}{C_A} = \frac{(V/E_o^*)}{4Z} R^2 . \exp \frac{-(V/E_o^*)}{4Z} r^2 \quad (2)$$

This solution includes a simplification possible when $Z > 5R$.

The problem of heat transfer in packed beds is complicated by the existence of transfer mechanisms other than those of diffusion and by the fact that heat is also transferred through the confining walls of the bed. The first difficulty can be circumvented by defining a total diffusivity, usually expressed as a total or effective thermal conductivity, which includes the rates of heat transfer due to molecular conduction, radiation, particle-to-fluid transfer, and natural convection, in addition to eddy conduction. Under these conditions Equation (1) becomes

$$\frac{\partial t}{\partial z} = \frac{k_e}{G_o C_p} \left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2} \right) \quad (3)$$

The term $k_e(\partial^2 t/\partial z^2)$ represents the heat transferred by diffusion in the direction of flow. Except at very low velocities this quantity is small compared with that carried by mass flow of the fluid and can be neglected. Thus

$$\frac{\partial t}{\partial z} = \frac{k_e}{G_o C_p} \left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial t}{\partial r} \right) \quad (4)$$

Radial temperature traverses have shown the existence of very steep temperature gradients next to the wall when heat is being transferred through the wall. This indicates an added resistance to heat transfer at the wall and has given rise to the concept of a wall heat transfer coefficient for a packed bed. Coberly and Marshall defined this coefficient as

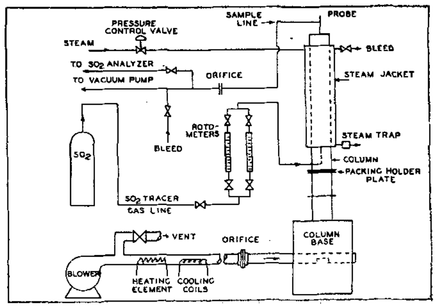

FIG. 1. FLOW DIAGRAM OF EXPERIMENTAL EQUIPMENT.

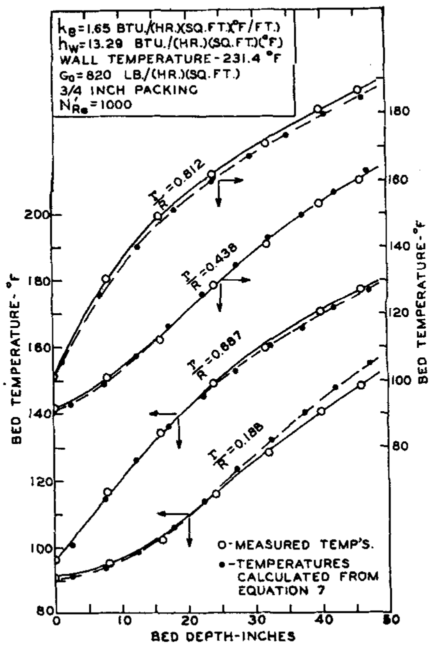

FIG. 2. COMPARISON OF MEASURED TEMPERATURE GRADIENTS WITH THOSE CALCULATED FROM EQUATION (7).

$$-k_e \left(\frac{\partial t}{\partial r} \right)_{r=R} = h_w (t_w - t_R) \quad (5)$$

EXPERIMENTAL EQUIPMENT

Eddy mass diffusion measurements were made by injecting a small stream of a foreign gas into the center of the fluid stream at the base of the packing and measuring the resulting concentration profiles at a given bed depth. Heat transfer measurements were made in a steam-heated, fixed-bed heat exchanger. Radial-temperature profiles were measured at several bed depths for a given flow rate by means of a high-velocity thermocouple probe placed immediately above the packing. Air was used as the main stream fluid, and sulfur dioxide as the tracer gas.

A schematic diagram of the experimental equipment is shown in Figure 1. The column was constructed from a 66-in. length of standard 8-in., Schedule-20, steel pipe, with a bolt

TABLE 1.—PHYSICAL PROPERTIES OF GLASS SPHERES

Nominal diameter, in.	Average diameter, ft.	Absolute density, lb. mass/cu. ft.	Thermal conductivity, B.t.u./(hr.)(sq. ft.)(°F./ft.)	D_p/D_t
1/2	0.0437	151.6	0.60	0.065
3/4	0.0596	151.6	0.60	0.088

TABLE 2.—EFFECT OF BED DEPTH ON WALL HEAT TRANSFER COEFFICIENT

N_{Re}: 1,000 Wall temperature: 231.4° F.
G_o'': 820 lb./(hr.) (sq. ft.) ¾ in. spherical packing

Incremental bed depth, in.	h_w
0–8	11.95
8–16	14.00
16–24	13.00
24–32	13.25
32–40	14.52
40–46	11.72
Over-all	13.29

flange welded to its lower end. This was jacketed with a 4-ft. section of 12-in. pipe, starting at a point 12 in. above the column flange. The lower unjacketed portion constituted an entrance section which allowed the fluid to establish a flow pattern before entering the measuring section. Eleven equally spaced No. 24 copper-constantan thermocouples were installed in the jacketed portion of the tube wall for measuring the tube-wall temperature. Steam was admitted into the jacket through its upper jacket flange, and condensate was withdrawn through the lower jacket flange. Steam pressure in the jacket was controlled by a Fisher diaphragm controlled valve, type 57T, and an associated pilot unit. In all cases the pressure was controlled at 8 lb./sq. in. gauge.

The base of the column was a 55-gal. steel drum which had a 12-in. section of 8-in. pipe welded in the center of the top and was pierced across the midsection by the air-inlet line. To aid in emptying the column, the base was joined to the column through an easily removable 12-in. section of similar pipe bolted in place. The packing was supported by a removable grid bolted to the column flange.

Air was supplied from a 10-hp. Roots-Connorsville cycloidal blower, rated at 200 std.cu.ft./min., with a back pressure of 7 lb./sq. in. gauge. Standard 2-in. steel pipe served as the air line. The air to the column passed over an internal electrical heating coil and then over copper cooling coils to permit accurate temperature control. Metering was accomplished by means of a set of orifice flanges fitted with interchangeable orifice plates.

A modified high-velocity thermocouple probe was used to make temperature measurements. This consisted of seventeen 3/32-in. I.D. copper tubes 3 in. long; all were mounted in a header on 1/2-in. centers except the two extreme tubes, which were mounted on 3/8-in. centers. This permitted temperature measurements to within 3/16-in. of the wall. Each copper tube contained a 24-gauge copper-constantan thermocouple, the bead of which was kept from contacting the tube by means of a short length of ceramic tubing. When the probe was in operation, air was drawn through the tubes at a velocity equal to that of the air in the bed to minimize mixing effects due to drawing in air at too high a rate. The air in the tube was accelerated in the vicinity of the thermocouple bead by the constriction formed by the bead and ceramic tube. A Leeds and Northrup portable precision potentiometer was used to measure thermocouple electromotive forces.

Sulfur dioxide of 99.9% purity, was used as the tracer gas in mass diffusion runs. It was piped from the cylinder to a 1/4-in. stainless steel needle valve and then through one of two rotameters to the injection tube, an L-shape piece of 1/8-in. galvanized pipe passing through the column wall 3 in. above the flange and opening into the center of the column on a level with the lower steam-jacket flange, which constituted the beginning of the measuring section.

The mass-diffusion sampling probe consisted of nine sampling tubes, one aligned along the central axis of the column and the others spaced 3/4, 1 1/2, 2 1/4, and 3 in., respectively, from the center along one radius and 1 1/8, 1 7/8, 2 5/8, and 3 3/16 in., respectively, from the center tube along the other radius. The upper ends of the sampling tubes were connected by means of copper tubing to a bank of 1/4-in. gate valves, all of which were connected to a common sampling line. Concentrations of sulfur dioxide in the air were measured by means of a Thomas Autometer (*7, 18*).

PROCEDURE

Two sets of data were taken in the mass transfer part of the work: one with the bed at a uniform temperature and the other with a temperature gradient impressed across the bed.

For isothermal runs the bed was first packed to the desired depth and the sampling probe lowered into position. The blower was started, the barometer and air-temperature thermometer were read, and the air rate was adjusted to the proper value. Sulfur dioxide was admitted into the column at the base of the measuring section through the injection tube at a velocity just slightly less than the average linear velocity of the air in the bed interstices. The valve of the desired sampling tube was opened and the sample-line vacuum system and sulfur dioxide analyzer turned on. When steady state conditions were reached, the sulfur dioxide concentration was obtained from the Thomas Autometer. A different sampling tube was then connected in the line and the procedure repeated.

When runs were made with a temperature gradient across the bed, an identical procedure was followed except that before the tracer gas was admitted to the column the steam was turned on and the equipment allowed to attain a thermal steady state. The tracer gas was then turned on and the run proceeded as before.

For heat transfer measurements, the blower was started and the air flow adjusted to the approximate rate by means of a bleed valve. Steam was turned on and the pressure in the steam jacket adjusted to 8 lb./sq. in. gauge. The column was packed until the top of the packing was coincident with the lower steam-jacket flange. The air-line temperature was adjusted to 85°F. by means of heating and cooling coils, the barometer read, and the air rate adjusted to the exact value. The thermocouple probe was then lowered into the column until the tubes were at the level of the packing but not touching it. The vacuum system was started, and flow in the vacuum line adjusted to the proper rate. After steady state was achieved, the thermocouple readings were recorded. The probe was turned through an angle of 45° and another set of thermocouple readings taken. This was repeated twice more, giving eight readings at each radial distance, which were averaged arithmetically to give the reading at that position.

Readings were taken of the eleven thermocouples embedded in the tube wall. The probe was then removed and additional packing added. The same general sequence of operations was followed at each packing depth until the desired quantity of packing had been added.

CALCULATIONS

Mass Transfer. To apply Equation (2) to concentration-profile measurements, the equation can be written in the following form

$$\ln \frac{C}{C_A} = \frac{-(V/E_o^*)}{4Z} r^2 + \ln \frac{(V/E_o^*)}{4Z} R^2 \qquad (2A)$$

This equation is linear in $\ln(C/C_A)$ and r^2, and a plot of $\ln(C/C_A)$ vs. r^2 gives a straight line with a slope of $\frac{-(V/E_o^*)}{4Z}$ and an intercept of $\ln \frac{(V/E_o^*)}{4Z} R^2$. The value V/E_o^*

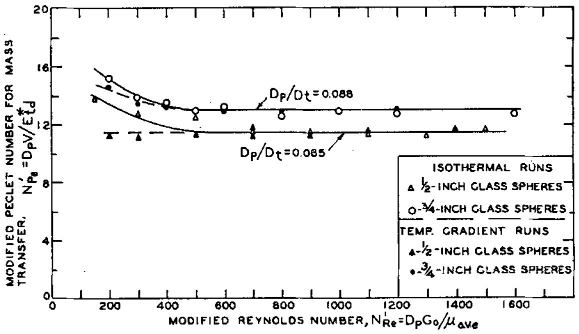

FIG. 3. CORRELATION OF MASS DIFFUSION DATA.

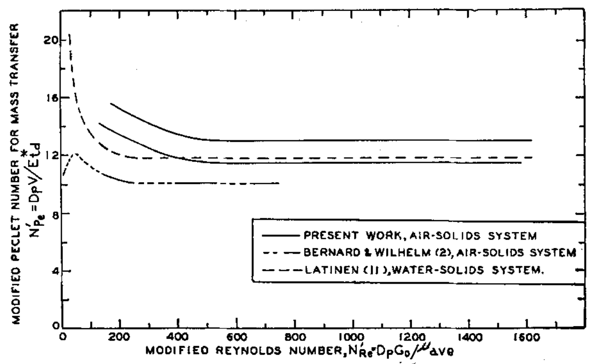

FIG. 4. COMPARISON OF ISOTHERMAL-EDDY MASS DIFFUSION DATA.

can be calculated from either quantity. Using the intercept, however, requires taking the antilogarithm of an experimentally determined value, a procedure that magnifies small errors. More consistent results were obtained by using the slope, which can be evaluated from Equation (2A) by least squares as

$$\frac{-(V/E^*_o)}{4Z} = \frac{\Sigma r^2\,\Sigma \ln (C/C_A) - a\,\Sigma r^2 \ln (C/C_A)}{(\Sigma r^2)^2 - a\,\Sigma r^4} \quad (6)$$

where a is the number of readings included in the summation.

Equations (2A) and (6) apply only to diffusion in the central core of the tube, that is, to concentration profiles in which the ratio of the maximum concentration to the concentration at the wall is 6:1 or more. Solutions for more complex cases have been presented by Bernard and Wilhelm (*2*), and Klinkenberg *et al.* (*10*).

Heat Transfer — Effective Thermal Conductivity. The values of k_e were determined by trial-and-error numerical solutions of Equation (4). If the substitution $\Gamma = \ln r$ is made, Equation (4) becomes

$$\Delta_z t = \frac{k_e}{G_oC_p} \cdot \frac{\Delta z}{(\Delta r)^2} \Delta_\Gamma^2 t \quad (7)$$

where $\Delta_z t$ refers to an increment of temperature in the longitudinal direction and $\Delta_r t$ represents a temperature increment in the radial direction. This equation can be solved by methods outlined in Sherwood and Reed (*16*), subject to the following boundary conditions:

1. At $z = 0$, $t = t_{inlet}$ for all values of r. It was not necessary to assume a uniform inlet temperature, as the inlet temperature distribution was measured.

TABLE 3.—EDDY MASS DIFFUSION DATA

Isothermal bed temperature: 90° F.
$D_{m_{90°F.}}$: 0.464 sq. ft./hr.

Average nonisothermal bed temperature: 140° F.
$D_{m_{140°F.}}$: 0.524 ft./hr.

Void fraction: 0.385

Modified Reynolds number, D_pG_o/μ_{avg}	Eddy mass diffusivity E_{td}^*, sq. ft./hr.	Modified Peclet number, D_pV/E_{td}^*
	1/2-in. packing—isothermal bed	
150	18.51	13.75
300	38.97	12.74
500	66.46	12.45
700	101.12	11.45
700	100.41	11.53
900	130.14	11.44
1,100	161.57	11.29
1,300	192.42	11.18
1,500	213.10	11.65
	1/2-in. packing—nonisothermal bed	
200	32.16	11.22
300	49.10	11.03
500	79.61	11.33
700	113.37	11.14
900	149.29	10.88
900	144.61	11.23
1,100	172.84	11.48
1,400	216.70	11.66
	3/4-in. packing—isothermal bed	
200	21.80	15.18
300	35.89	13.83
400	48.80	13.56
500	64.10	12.90
600	74.90	13.25
800	105.32	12.57
1,000	127.37	12.99
1,200	155.47	12.77
1,600	207.62	12.75
	3/4-in. packing—nonisothermal bed	
200	24.65	14.64
300	40.01	13.53
400	54.96	13.13
600	83.43	12.98
800	112.02	12.89
1,200	165.31	13.10

2. At $r = R$, for all values of z

$$\left(\frac{\Delta t}{\Delta \Gamma}\right)_{r=R} = \frac{(t_w - t_R)}{\dfrac{k_e}{h_w R}}$$

3. At $r = 0$, for all values of z

$$\left(\frac{\Delta t}{\Delta \Gamma}\right)_{r=0} = 0$$

This method presupposes a knowledge of the wall heat transfer coefficient, which can be calculated by the methods outlined below.

A value of k_e was assumed and a family of radial-temperature-gradient curves constructed by application of Equation (7), each curve being displaced from the previous one by the amount Δz. These were compared with the measured curves and the whole procedure repeated until a value of k_e was found that gave best agreement between calculated and measured temperature gradient curves. A typical plot of this type is shown in Figure 2.

Wall Heat Transfer Coefficient. Wall heat transfer coefficients were calculated by a heat balance for the column. If a uniform velocity distribution over the bed and constant fluid properties are assumed, the equation is

$$G_o (\pi R^2)\, C_p\, (t_{go} - t_{gi}) = h_w\, (2\pi R Z)\, (t_w - t_R)_{mean} \quad (8)$$

The average inlet and outlet gas temperatures were calculated from measured temperature profiles by the equation

$$\frac{t_{avg}}{460 + t_{avg}} = \frac{2}{R^2} \int_0^R \frac{tr}{460 + t} \cdot dr \quad (9)$$

The quantity t_R, which represents the temperature at the inner boundary of the wall film, was estimated by extrapolating the measured temperature profiles to the wall. The mean temperature difference $(t_w - t_R)$ was determined by plotting $(t_w - t_R)$ at each bed depth against the bed depth and the average ordinate found by graphical integration.

MASS TRANSFER DATA

Eddy mass diffusivities were correlated in terms of modified Peclet numbers, $D_p V / E^*_{td}$, and modified Reynolds numbers, $D_p G_o / \mu$, which are symbolized as N'_{Pe} and N'_{Re}, respectively. This plot is presented in Figure 3 and all eddy mass diffusion data are given in Table 3.

From Figure 3 it would appear that the modified Peclet number is also a function of the ratio D_p/D_t. This, however, is probably due to dissimilar packing orientation. At $D_p/D_t = 0.088$, values of the Peclet numbers obtained after successive bed repackings were as much as 15% below the curve shown and agreed quite well with the lower curve. This behavior would be expected since it is not possible to reproduce accurately the orientation of packing even though the void fraction can be reproduced well. If the bed were repacked a sufficient number of times, it is probable that the upper curve would be brought closer to the lower curve. In order to discern the effect of a temperature gradient, all data shown for a given packing size were made without repacking the bed.

The curves for the different packing sizes are similar in shape and show two distinct regions. At modified Reynolds numbers above about 500 the Peclet number is constant, indicating that the eddy diffusivity is directly proportional to the fluid velocity and particle diameter. Such behavior is characteristic of fully developed turbulence and agrees well with the "random walk" analysis of Baron and Latinen.

At Reynolds numbers below 500, fully developed turbulence for mixing no longer exists and the N'_{Pe} varies with the fluid velocity. The same type of behavior was observed by Bernard and Wilhelm and by Latinen (Figure 4).

The broken line of Figure 3 represents eddy-mass-diffusion measurements made in the presence of a temperature gradient. In the region of fully developed turbulence a temperature gradient appears to make no difference. At low Reynolds numbers, however, there is a difference although the effect is much more pronounced in the case of the smaller packing. This effect might be caused by mixing due to localized natural convection currents even though the over-all conditions are not conducive to natural convection currents involving large masses of fluid. Unfortunately, the data are not sufficient to permit any general conclusions, but they show the need for further study in this area.

The eddy-mass-diffusion data of Bernard and Wilhelm (*2*) and of Latinen (*11*) are shown in Figure 4 along with the present work. The data of Latinen were obtained in a 2-in. column packed with glass spheres with water as the main stream fluid. The values of D_p/D_t ranged from 0.02 to 0.16. Bernard and Wilhelm made eddy-mass-diffusion measurements for 3/8-in. alumina spheres in an 8-in. tube $(D_p/D_t = 0.047)$, with air as the fluid. The agreement of all the data is fairly good, particularly in view of the wide divergence in properties of the systems investigated.

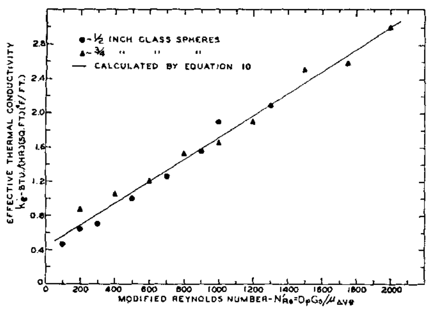

FIG. 5. CORRELATION OF EFFECTIVE THERMAL CONDUCTIVITIES.

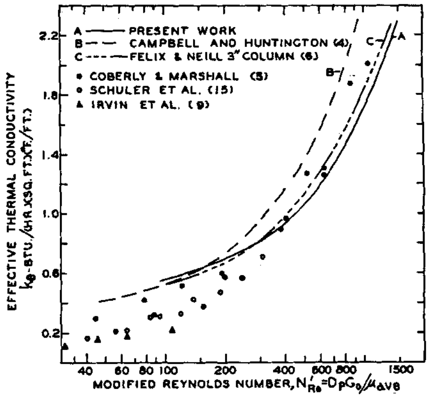

FIG. 6. COMPARISON OF EFFECTIVE THERMAL CONDUCTIVITY DATA.

HEAT TRANSFER DATA

Effective thermal conductivity values were correlated by the equation

$$k_e = 0.439 + 0.00129\, (D_p G_o / \mu_{avg}) \quad (10)$$

Figure 5 is a plot of this correlation. The average deviation of the experimental points from this line is 9.4%. All effective thermal conductivity values are summarized in Table 4.

The term μ_{avg} is the molecular viscosity of the fluid at the average bed temperature. Since the viscosity varied by less than 15% it was not possible to segregate the effect of this variable, and so an average value was used.

One of the assumptions made in deriving Equation (4) was that k_e did not vary with radial position. This was checked, and in almost all cases k_e was found to vary by less than 10% up to a distance of about one particle diameter of the wall.

The effective thermal conductivities are compared in Figure 6 with the data of other investigators. The abscissa is plotted on a logarithmic scale to afford a better distribution at low Reynolds numbers, where much of the comparative data lies. Curve *A* is a plot of Equation (10) of this work. The solid points represent the data of Coberly and Marshall (*5*) for 1/8- by 1/8-, 1/4- by 1/4-, and 3/8- by 1/2-in. Celite cylinders in a 5-in. tube. Curve *C* represents the correlation obtained by Felix and Neill (*6*) for glass spheres in a 3-in. tube. The agreement in each case is quite good.

Campbell and Huntington (*4*) measured effective thermal conductivities for both cylinders and spheres of several sizes and materials in 2-, 4-, and 6-in. tubes, using both air and natural gas as the flowing fluid. Curve *B* is a plot of their final correlation when air was the fluid. They obtained k_e values by graphically differentiating temperature-profile curves at several points along the axis of the bed and taking an average. Since these values were obtained in the region of minimum packing nonuniformities, they might be expected to be somewhat larger than k_e values averaged for the entire bed. They also found k_e to be directly proportional to the molecular thermal conductivity of the fluid.

The data obtained by Schuler *et al.* (*15*) and by Irvin *et al.* (*9*) for 1/8-, 3/16-, and 1/4-in. cylindrical alumina pellets packed in a 2-in. tube fall somewhat below the other results. As mentioned before, they accounted for wall effects by decreasing k_e in the vicinity of the wall. The points of Figure 6 represent effective thermal conductivity values averaged for the whole bed. As these incorporate the wall resistance in this instance, they would be expected to be less than k_e values free of wall effects.

To compare Peclet numbers for heat and mass transfer in a packed bed, eddy thermal conductivity values (k_{td}) must be known. Since eddy diffusion results from turbulence generated in the packing interstices by the fluid flowing around the packing, the rate of transfer attributable to this mechanism would be zero at zero rate of flow, and k_{td} would also be zero. If the $k_e = N'_{Re}$ curve is extrapolated to $N'_{Re} = 0$, the intercept value of 0.439 in Equation (10) can be regarded as the approximate sum of the rates of heat transfer due to molecular conduction, radiation, possible localized natural convection, and particle-to-fluid heat transfer. This value is in reasonable agreement with the value of 0.31 reported by Campbell and Huntington and the value of 0.365 of Felix and Neill. k_{td} values can be estimated from k_e values by the equation

$$k_{td} = k_e - 0.439 \qquad (11)$$

The modified Peclet number for heat transfer is

$$N'_{Pe_h} = \frac{D_p G_o C_p}{k_{td}} \qquad (12)$$

Substituting Equations (10) and (11) into (12) yields

$$N'_{Pe_h} = \frac{C_p \mu_{avg}}{0.00129} \qquad (13)$$

A plot of modified Peclet numbers for heat and mass transfer is

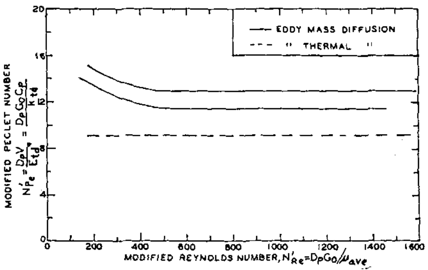

Fig. 7. Comparison of Peclet Numbers for Heat and Mass Transfer.

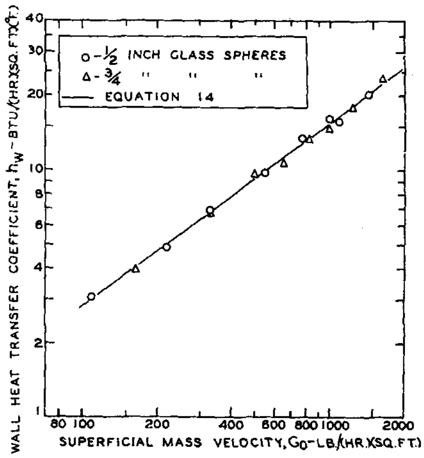

Fig. 8. Correlation of Wall Heat Transfer Coefficients.

Table 4.—Effective Thermal Conductivity and Wall Heat Transfer Coefficient Data

Average bed temperature : 150° F.

$\mu_{avg\,(150°F.)}$: 0.0488 lb./(hr.) (ft.)

Superficial mass velocity, G_o, lb./(hr.) (sq. ft.)	Modified Reynolds number, D_pG_o/μ_{avg}	Effective thermal conductivity, k_e, B.t.u./(hr.) (sq. ft.) (° F./ft.)	Wall heat transfer coefficient, h_w, B.t.u./(hr.) (sq. ft.)(°F.)
	1/2-in. packing		
109	98	0.47	3.05
218	195	0.65	4.83
327	293	0.70	6.92
545	488	1.00	9.73
764	684	1.26	13.58
982	879	1.56	16.10
1,092	978	1.90	15.74
1,420	1,272	2.09	20.10
	3/4-in. packing		
164	200	0.87	3.95
328	400	1.05	6.80
491	600	1.20	9.59
655	800	1.53	10.56
820	1,000	1.65	13.29
983	1,200	1.90	14.70
1,228	1,500	2.50	17.92
1,433	1,750	2.58	19.68
1,638	2,000	3.00	23.30

shown in Figure 7. The modified Peclet numbers for heat transfer calculated by this method are about 25% below those for mass transfer. This is not unexpected since this method of calculating modified Peclet numbers for heat transfer is based on the assumption that the rate of heat transfer due to all mechanisms except eddy diffusion is constant. It is probable that this value is not constant but increases slightly with the Reynolds number. If the assumption is made that the value is a small linear function of the Reynolds number, the agreement between Peclet numbers for heat and mass transfer would be even better.

WALL HEAT TRANSFER COEFFICIENT

Wall heat transfer coefficients were measured over a range of Reynolds numbers from 98 to 2,000. The data are given in Table 4, and the final correlation is shown in Figure 8. The equation of this curve is

$$h_w = 0.090\, G_o^{0.75} \qquad (14)$$

The average deviation of the experimental points from this line is 3.1%.

The effect of bed depth on the wall heat transfer coefficient was investigated by calculating h_w values for different incremental bed depths. The results of a typical run are given in Table 2. No trend with bed depth is apparent; instead, a constant wall heat transfer coefficient is indicated. The scattering in values may be attributed largely to the fact that for a shallow section of bed a small error in calculating the average inlet or outlet air temperature is magnified in the temperature-difference term of the heat balance and greatly affects the local wall heat transfer coefficient. When an over-all heat balance is used, however, the effect of an error of this type is considerably lessened. All h_w values were calculated from over-all heat balances.

Equation (14) shows the wall heat transfer coefficient to be proportional to the 0.75 power of the superficial mass velocity. In turbulent flow in an open channel the heat transfer coefficient is proportional to $G_o^{0.8}$. The similarity in velocity dependence is rather surprising in view of the difference in the two systems and suggests a possible similarity in flow conditions and velocity gradients near the wall.

A sphere contacts the wall at only one point, and the area of contact is small. Furthermore the portions of the spheres adjacent to the wall present little form drag to the flow of fluid along the wall. This type of arrangement would present a minimum of disturbance to the fluid flowing along the wall and would tend to promote channeling. With channeling, flow in this area would not be unlike flow in an annulus, and a similar velocity dependence of h_w might be expected.

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NOTATION

- C = concentration of diffusing material, lb./cu.ft.
- C_A = mixed average effluent concentration of diffusing material, lb./cu. ft.
- C_p = fluid heat capacity at constant pressure, B.t.u./(lb.) (°F.)
- D_m = molecular diffusivity, sq.ft./hr.
- D_p = packing-particle diameter, ft.
- D_t = inside diameter of containing tube, ft.
- E^*_o = total diffusivity based on void area, sq.ft./hr.
- E^*_{td} = eddy diffusivity based on void area, sq.ft./hr.
- f = factor representing concentration of diffusing material for mass transfer and temperature in heat transfer
- G_o = superficial mass velocity based on empty tube area, lb./(hr.) (sq.ft.)
- h_w = wall heat transfer coefficient B.t.u./(hr.) (sq.ft.) (°F.)
- k_e = effective thermal conductivity of bed based on void plus nonvoid area, B.t.u./(hr.) (sq.ft.) (°F./ft.)
- k_{td} = eddy conductivity of the fluid phase due to turbulent mixing, B.t.u./(hr.) (sq.ft.) (°F./ft.)
- N'_{Pe} = modified Peclet number for mass transfer, D_pV/E^*_{td}
- N'_{Pe_h} = modified Peclet number for heat transfer, $D_pG_oC_p/k_{td}$
- N'_{Re} = modified Reynolds number, D_pG_o/μ_{avg}
- R = radius of containing tube, ft.
- r = radial coordinate in cylindrical coordinates
- t = temperature, °F.
- t_{gi} = average inlet air temperature, °F.
- t_{go} = average outlet air temperature, °F.
- t_R = fluid temperature at inner boundary of wall film, °F.
- V = local velocity in longitudinal direction, ft./hr.
- Z = packing depth, ft.
- z = longitudinal or axial coordinate in cylindrical coordinates

Greek Symbols

- ε = void fraction
- ρ = density of fluid, lb./cu.ft.
- $\Gamma = \ln r$
- μ_{avg} = molecular viscosity of fluid at average bed temperature, lb./(ft.) (hr.)

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